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Superstrings and WZNW Models

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We give a brief review of our approach to the quantization of superstrings. New is a covariant derivation of the measure at tree level and a path integral formula for this measure.

1 A brief review

In a series of papers [1] we have presented a new approach to the old problem of the covariant quantization of the superstring. The fields of the classical superstring are $x^m(z, \bar{z})$ and two ($i=1,2$) spacetime spinors $\theta^{\alpha i}(z, \bar{z})$ where $m = 0, \dots, 9$ is a vector index and $\alpha = 1, \dots, 16$ the index of a real chiral spinor. A natural choice as action might seem to be the supersymmetric line element

$$S_1 = -\frac{1}{2} \int d^2x \Pi_\mu^m \Pi_m^\mu, \quad (1)$$

where $\Pi_\mu^m = \partial_\mu x^m - i \sum_i \theta^{\alpha i} \gamma_{\alpha\beta}^m \partial_\mu \theta^{\beta i}$ with $\gamma_{\alpha\beta}^m = \gamma_{\beta\alpha}^m$ real Dirac matrices.

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It is invariant under the rigid supersymmetry transformations $\delta_\epsilon x^m = i\epsilon^i \gamma^m \theta^i$ and $\delta_\epsilon \theta^{\alpha i} = \epsilon^{\alpha i}$. However, there is no kinetic term for $\theta^{\alpha i}$. One can add a Wess-Zumino term [2]

$$S_{WZ} = \int \sum_i (-)^i d\theta^i \gamma^m d\theta^i \Pi_m, \quad (2)$$

It is separately supersymmetrically invariant, and $S_1 + S_{WZ}$ has the (in)famous local κ symmetry which allows one to choose the light-cone gauge in target space. Manifest Lorentz covariance is then lost. So far the covariant quantization of this action has remained an open problem.

Our approach begins with an idea of Berkovits [3] who started from the BRST charge

$$Q_B = \oint dz \lambda^\alpha d_\alpha, \quad d_\alpha = p_\alpha + i\partial x^m (\gamma_m \theta)_\alpha + \frac{1}{2} (\gamma^m \theta)_\alpha (\theta \gamma_m \partial \theta). \quad (3)$$

The λ^α are commuting ghosts. We are now on the Euclidean worldsheet with light-cone derivatives $\partial = \partial_\sigma - i\partial_\tau$ and restrict our attention to only one θ^α . The OPE's $x^m(z)x^n(w) \sim -\eta^{mn} \ln(z-w)$ and $p_\alpha(z)\theta^b(w) \sim \delta_\alpha^\beta (z-w)^{-1}$ can be used to show that

$$d_\alpha(z)d_\beta(w) \sim 2i \frac{\gamma_{\alpha\beta}^m \Pi_m}{(z-w)}. \quad (4)$$

The equation $d_\alpha = 0$ yields the conjugate momentum of θ^α as obtained from the action $S_1 + S_{WZ}$ [1], and is the only ingredient we retain of the Green-Schwarz model [2]. We build upon this d_α a whole new structure, and we expect that the results we obtain in the end will be equivalent to whatever one would call the covariant quantum superstring.

From (4) it follows that $Q_B^2 = 2i \oint \lambda \gamma^m \lambda \Pi_m$. Clearly, if λ are so-called pure spinors, which satisfy by definition $\lambda \gamma^m \lambda = 0$, we obtain a nilpotent BRST charge [3]. These λ^α must be complex in order to have a nontrivial solution of $\lambda \gamma^m \lambda = 0$.

In [1] we have relaxed the constraint of pure spinors by adding a Lagrange multiplier term to Q_B of the form $-i \oint \beta_m \lambda \gamma^m \lambda$. Now λ^α need not longer be complex, and we take it real, just like θ^α . In fact we first determined the superalgebra generated by d_α ; it contains in addition to Π_m also $\partial \theta^\alpha$, and in the OPE's one finds terms with single poles and double poles. Denoting

these three operators by $J_M^g = \{\Pi_m, -id_\alpha, \partial\theta^\alpha\}$, the OPE has the form

$$J_M^g(z)J_N^g(w) \sim \frac{J_P^g f_{MN}^P}{(z-w)} - \frac{\mathcal{H}_{MN}}{(z-w)^2}. \quad (5)$$

The BRST charge is then given

$$Q = - \oint dz (J_M^g + \frac{1}{2} J_M^{gh}) c^M \quad (6)$$

where $c^M = \{\xi^m, \lambda^\alpha, \chi_\alpha\}$ are the ghosts, and the BRST ghost current J_M^{gh} depends also on b_M , where $b_M = \{\beta_m, w_\alpha, \kappa^\alpha\}$ are the antighosts. The susy generator $q_\alpha = p_\alpha - i\partial x^m (\gamma_m \theta)_\alpha - \frac{1}{6} (\gamma_m \theta)_\alpha (\theta \gamma^m \partial \theta)$ transforms θ^α into ϵ^α and x^m into $i\epsilon \gamma^m \theta$, and anticommutes with the covariant derivative d_α .

The three operators J_M^g define a non-semisimple super Lie algebra [4], but the invariant metric \mathcal{H}_{MN} is still nonsingular. The generators of this algebra are $(P_m, Q_\alpha, K^\alpha)$ where K^α is a fermionic central charge. In a coset approach K^α corresponds to new anticommuting coordinates ϕ_α which form with the usual θ^α a bigger, but little understood superspace [4].

The ghost currents $J_M^{gh} = \{2\kappa \gamma_m \lambda, 2\xi^m (\gamma_m \kappa) + 2i\beta^m (\gamma_m \lambda, 0\}$ satisfy the same OPE's as J_M^g but without double poles. It then follows as usual that the BRST charge Q is classically (using single contractions) nilpotent, but at the quantum level it ceases to be nilpotent due to the double poles. For that reason we have added, in the spirit of gauged WZNW models [5], a new triplet of currents J_M^h which have the same OPE's as J_M^g except that the sign of the double poles is opposite. The BRST charge is obtained by replacing J_M^g by $J_M^g + J_M^h$. Fatal double poles in contractions now cancel and this extended Q is nilpotent. However, we have doubled all coordinates: in addition to x^m, θ^α and p_α we also have x_h^m, θ_h^α and p_α^h .

We have shown in [1] that a WZNW action based on the currents J_M^g, J_M^{gh} and J_M^h , but in terms of ϕ_α instead of p_α , reduces to the free field action which in turn determines the propagators, provided one expresses $\partial_z \phi_\alpha$ in terms of $p_{z\alpha}$ by $i\partial_z \phi_\alpha = q_{z\alpha}$. Thus all our previous work turns out to have been based on a WZNW model. We work from now on in the formulation without ϕ_α but with p_α .

This model has a very interesting conformal field theory. The BRST charge is

$$\begin{aligned} j_z^B = & -\lambda^\alpha (-id_{z\alpha} + J_{z\alpha}^h) - \xi^m (\Pi_{zm} + J_{zm}^h) - \chi_\alpha (\partial_z \theta^\alpha + \\ & + J_z^{h,\alpha}) - 2\xi^m \kappa_z \gamma^m \lambda - i\beta_{zm} (\lambda \gamma^m \lambda). \end{aligned} \quad (7)$$

The energy-momentum tensor T_{zz} is

$$T_{zz} = -\frac{1}{2}\partial_z x^m \partial_z x_m - p_{z\alpha} \partial_z \theta^\alpha - \beta_{zm} \partial_z \xi^m - \kappa_z^\alpha \partial_z \chi_\alpha - w_\alpha \partial_z \lambda^\alpha + \frac{1}{2} J_{zm}^h J_z^{h,m} + i J_{z\beta}^h J_z^{h,\beta}. \quad (8)$$

and the total conformal charge vanishes: $c = 10 - 32 - 20 + 32 + 32 + 10 - 32 = 0$. The ghost current is

$$j_z^{gh} = -\beta_{zm} \xi^m - \kappa_z^\alpha \chi_\alpha - w_{z\alpha} \lambda^\alpha. \quad (9)$$

Since the anomaly in the OPE $j_z^{gh}(z)j_w^{gh}(w) = c_j/(z-w)^2$ of the ghost current with itself is not zero but given by $c_j = -22$, while $T_{zz}(z)j_w^{gh}(w) = 22/(z-w)^3 + j_z^{gh}(z)/(z-w)^2$, this superconformal algebra seems to be a twisted version of an $N = 2$ superconformal algebra. The BRST current is nilpotent $j_z^B(z)j_w^B(w) \sim 0$. There is also a fermionic current B_{zz} which is the dual of the BRST current

$$B_{zz} = \frac{1}{2}(J_M^g - J_M^{gh})b_N \mathcal{H}^{NM}. \quad (10)$$

It has spin 2 and squares to $B_{zz}(z)B_{ww}(w) \sim \frac{F_{www}(w)}{(z-w)}$. The current F_{zzz} is not only BRST closed but also BRST exact

$$j_z^B(z)\Phi_{www}(w) \sim \frac{F_{www}}{(z-w)}, \quad \Phi_{zzz} = \frac{-i}{2}\beta_z^m \kappa_z \gamma^m \kappa_z \equiv b_Q b_P b_R f^{RPQ}. \quad (11)$$

The six currents $j_z^B, B_{zz}, j_z^{gh}, T_{zz}, F_{zzz}$ and Φ_{zzz} generate a closed algebra. However, it is not an ordinary $N=2$ superalgebra, but rather a deformation called a Kazama algebra [7].

Following [8] one can recover an ordinary $N=2$ superconformal algebra by adding a topological gravity quartet, also called a Koszul quartet [6]. It contains the usual anticommuting ghost pair (b_{zz}, c^z) and a commuting spin $(2, -1)$ counterpart (β_{zz}, γ^z) . The ghost numbers are $(-1, 1)$ and $(-2, 2)$ respectively. The currents of this model form an ordinary $N = 2$ superconformal algebra, so without Φ_{zzz} and F_{zzz} currents. However, if one adds the currents of the Koszul model to the currents of the WZNW model, and modifies B_{zz} appropriately, one ends up with an ordinary $N = 2$ superconformal algebra for the combined system.

2 Tree level measure

In this section, we give a derivation of the measure in our formalism. For that purpose, we follow the idea that the tree level action for the target space theory (the string field theory action) is determined by the so-called BV measure. Assuming that the target space action is of Chern-Simons type [9], one has

$$S_{D=10} = \int d\mu \left(\Phi Q \Phi + \frac{2}{3} \Phi \star \Phi \star \Phi \right), \quad (12)$$

where Φ is the string field (see (13)) and the product \star denotes the usual matrix multiplication and tracing over $U(n)$ matrices. The integral in the above equation is the path integral over the zero modes of $x^m, \theta^\alpha, \lambda^\alpha, \xi^m$ and χ_α (and ξ , see below) which remains after one has performed the path over all nonzero modes. In order to perform the integration, we have to establish the measure $d\mu$. Since Φ has ghost number one, the ghost number of the Lagrangian is three. Therefore, we have to impose that the measure has ghost number -3 (notice that, in contrast to Berkovits' work we have more ghosts present, therefore there are several possible combinations).

A second requirement is the BRST invariance of the measure. To construct the measure it is convenient to consider a ghost number 3 polynomial $\Omega^{(3)}$ which is invariant under super-Poincaré transformations (up to Q -exact terms) such that

$$\langle \Omega^{(3)} \rangle \equiv \int d\mu \Omega^{(3)} = 1. \quad (13)$$

The operator $\Omega^{(3)}$ represents the Poincaré dual to the measure (see for example Bott and Tu [10]). The condition of BRST invariance of $\Omega^{(3)}$ is therefore given by $\int d\mu \{Q, \Omega^{(3)}\} = 0$.

The measure $d\mu$ itself should also be BRST invariant. We implement this requirement by imposing the condition that if $\Omega^{(3)} = \{Q, \mathcal{K}\}$ where \mathcal{K} is any polynomial with ghost number 2, then

$$\int d\mu \{Q, \mathcal{K}\} = 0. \quad (14)$$

Since the action in (13) should reproduce the super-Yang-Mills action in $d = (9, 1)$ dimensions, we can choose a given coupling to normalize the integral in (13). For example the gluon-gluino-gluino coupling $f_{abc} v_\mu^a(x) \psi^{b\alpha} \gamma_{\alpha\beta}^m \psi^{c\beta}$

may provide a convenient normalization. A generic vertex reads

$$\begin{aligned}\Phi = & \lambda^\alpha A_\alpha + \xi^m A_m + \chi_\alpha W^\alpha + \\ & + \xi \left(\xi^m \xi^n F_{mn} + \xi^m \chi_\alpha F_m^\alpha + \lambda^\alpha \chi_\beta F_\alpha^\beta + \chi_\alpha \chi_\beta F^{\alpha\beta} \right),\end{aligned}\quad (15)$$

where the superfields $A_\alpha, \dots, F^{\alpha\beta}$ depend only the combinations $x^m - x_h^m$ and so on. (The other combinations are excluded by imposing the condition $B_0\Phi = 0$ on the vertex operator.) We also assume that all superfields are Lie algebra valued. The field ξ is obtained by the bosonization of the superghosts $\gamma^z = \eta e^\phi$ and $\beta_{zz} = \partial\xi e^{-\phi}$ of the twisted topological Koszul quartet.

By expanding the superfields A_m and W^α one finds that the first coefficients coincide with the gluon field v_m^a and with the gluino $\psi^{a\alpha}$, respectively. Therefore, a candidate for $\Omega^{(3)}$ is given by

$$\Omega^{(3)} \sim \gamma^{m,\alpha\beta} \chi_\alpha \xi_m \chi_\beta. \quad (16)$$

because substitution into (11) and (10) reproduces the gluon-gluino-gluino coupling. Notice that this candidate for $\Omega^{(3)}$ has the correct ghost number, and also from a conformal point of view it has the right properties, namely it is a scalar, just like the corresponding element $\Omega_{bos}^{(3)} = c\partial c\partial^2 c$ of the bosonic string. The form of $\Omega^{(3)}$ and equation (13) imply that

$$\begin{aligned}\langle (\xi^m A_m)(z_1)(\chi_\alpha W^\alpha)(z_2)(\chi_\alpha W^\alpha)(z_3) \rangle & \rightarrow \\ \rightarrow \int d^{10}x f_{abc} v_m^a \psi^{b\alpha} \psi^{c\beta} \langle \xi_m \chi_\alpha \chi_\beta \rangle & = \int d^{10}x f_{abc} v_m^a \psi^{b\alpha} \psi^{c\beta} \gamma_{\alpha\beta}^m.\end{aligned}\quad (17)$$

However, $\Omega^{(3)}$ in (16) fails to be BRST invariant. To repair it, we consider the following two combinations of ghosts

$$\hat{\xi}^m = \xi^m + \lambda^\alpha \gamma_{\alpha\beta}^m \theta^b, \quad \hat{\chi}_\alpha = \chi_\alpha - 2i\xi^m \gamma_{m\alpha\beta} \theta^\beta + \frac{4i}{3} \gamma_{\alpha\beta}^m \theta^\beta \lambda^\gamma \gamma_{m\gamma\delta} \theta^\delta \quad (18)$$

which are invariant under BRST transformations. Then we have finally

$$\begin{aligned}\Omega^{(3)} & = \gamma_{\alpha\beta}^m \hat{\chi}_\alpha \hat{\xi}_m \hat{\chi}_\beta \\ & = (\chi \gamma^m \chi) \xi_m + 2\xi^m \xi^n \chi \gamma_{mn} \theta + \xi^m (\chi \gamma_m \gamma_n \theta) (\lambda \gamma^n \theta) + (\lambda \gamma^n \theta) (\chi \gamma_n \chi) \\ & + (\lambda \gamma^m \theta) (\lambda \gamma^n \theta) (\chi \gamma_{mn} \theta) + \xi^m \xi^n \xi^r \theta \gamma_{mnr} \theta + \xi^m \xi^n (\lambda \gamma^r \theta) \theta \gamma_{mnr} \theta \\ & + \xi^m (\lambda \gamma^n \theta) (\lambda \gamma^r \theta) \theta \gamma_{mnr} \theta + (\lambda \gamma^m \theta) (\lambda \gamma^n \theta) (\lambda \gamma^r \theta) \theta \gamma_{mnr} \theta\end{aligned}\quad (19)$$

where the first term reproduces the combination needed to fix the coupling of the gluon-gluino-gluino vertex. More important is the observation that

the last term in $\Omega^{(3)}$ is exactly the combination discussed in [3]. Substitution of this term with pure spinors into (12) and (11) reproduces the complete super-Yang-Mills theory, and the measure is in this case also BRST invariant. In our covariant approach the λ 's are not constrained to be pure spinors, and then we need all terms in (19) for BRST invariance.

In order, to fix the integration, one has to assume that a particular monomial $\Omega_i^{(3)}$ of $\Omega^{(3)}$ has the property $\int d\mu \Omega_i^{(3)} = 1$. One choice is obviously the last term, as we have learnt from [3]. Another choice is to fix the first term. However, we can also impose that another combination of terms gives the expected result.

3 Path Integral formula for the measure

In order to show that the ghosts and antighosts of the present formulation are emerging from the Berkovits formulation, we derive here a path integral formula for the measure (19) assuming that only the last piece in (19) (the so-called Berkovits term) $\Omega^{(3)} = (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^r\theta)\theta_{\gamma_{mnr}}\theta$ contributes to (13). We shall discuss in section 4 that this is indeed the case, but that at the same time we can continue with 16 unconstrained and real λ^α . Therefore, we write

$$1 = \int d^{16}\theta d^{16}\lambda \mu(\lambda, \theta) [(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^r\theta)\theta_{\gamma_{mnr}}\theta]. \quad (20)$$

and we want to compute $\mu(\lambda, \theta)$. At the first step, we notice that in order that the integral over the pure spinor fields λ is well defined the measure should contain a Dirac delta function to render the integration over the λ 's finite: $\mu(\lambda, \theta) = \delta^{16}(\lambda)\mu(\lambda, \theta)'$. The delta function in the measure is also needed for compensating the ghost number of $d^{16}\lambda$. In order that the measure $\mu'(\lambda, \theta)$ projects out three components of the pure spinor, it should have the form

$$\mu(\lambda, \theta) = \delta^{16}(\lambda)\mu^{(\alpha\beta\gamma)}(\theta)\partial_{\lambda^\alpha}\partial_{\lambda^\beta}\partial_{\lambda^\gamma}. \quad (21)$$

One needs exactly three λ -derivatives to take into account the ghost number.

The next step is to represent the delta function by an integral over variables w_α which we identify with our antighosts.⁴ The three derivatives be-

⁴This needs further study because the antighosts $w_{z\alpha}$ are vectors on the worldsheet which do not have zero modes on a genus zero Riemann surface.

come then three components of w

$$1 = \int d^{16}\theta d^{16}\lambda d^{16}w e^{i w_\alpha \lambda^\alpha} [\mu^{(\alpha\beta\gamma)}(\theta) w_\alpha w_\beta w_\gamma] \Omega^{(3)}. \quad (22)$$

The integration $d^{16}\theta$ should be saturated by the θ 's present in $\mu^{(\alpha\beta\gamma)}(\theta)$ and those present in $\Omega^{(3)}$. Knowing that in the latter there are 5 θ 's, we obtain

$$\mu^{\alpha\beta\gamma}(\theta) = \mu_{[\rho_1 \dots \rho_{11}]}^{(\alpha\beta\gamma)} \theta^{\rho_1} \dots \theta^{\rho_{11}}, \quad (23)$$

where $\mu_{[\rho_1 \dots \rho_{11}]}^{(\alpha\beta\gamma)}$ are numerical constants. Using the following formula for Berezin integrals

$$\theta^{\alpha_1} \dots \theta^{\alpha_{n-l}} = \frac{(n-l)!(-i)^l}{l!} \epsilon^{\alpha_1 \dots \alpha_{n-l} \rho_{n-l+1} \dots \rho_n} \int d^n p (p_{\rho_{n-l+1}} \dots p_{\rho_n}) e^{i p_\alpha \theta^\alpha}, \quad (24)$$

where p_α are 16 fermionic new variables and $\epsilon^{\alpha_1 \dots \alpha_{16}}$ is the invariant tensor of $Spin(9, 1)$, we obtain

$$1 = \int d^{16}\theta d^{16}\lambda d^{16}w e^{i \lambda^\alpha w_\alpha + i p_\alpha \theta^\alpha} [(p\gamma^m w) (p\gamma^n w) (p\gamma^r w_\gamma) (p\gamma_{mnr} p)] \Omega^{(3)}. \quad (25)$$

The contractions in (25) with the invariant tensors are dictated by Lorentz invariance. In particular we have

$$\mu_{[\rho_1 \dots \rho_{11}]}^{(\alpha\beta\gamma)} = \epsilon_{\alpha_1 \dots \alpha_5 \rho_1 \dots \rho_{11}} \gamma^{m, \alpha\alpha_1} \gamma^{n, \beta\alpha_2} \gamma^{r, \gamma\alpha_3} \gamma_{mnr}^{\alpha_4 \alpha_5}. \quad (26)$$

To simplify further the integral, we introduce 10 anticommuting variables ξ^m which correspond exactly to our ghost fields $\xi^m(z)$ discussed in the previous sections. We can then rewrite (25) as follows

$$1 = \int d^{16}\theta d^{16}\lambda d^{16}w d^{10}\xi e^{i \lambda^\alpha w_\alpha + i p_\alpha \theta^\alpha + \xi^m p \gamma_m w} \times \\ \times [\epsilon_{m_0 \dots m_9} \xi^{m_0} \dots \xi^{m_6} (p \gamma^{m_7 m_8 m_9} p)] \Omega^{(3)}. \quad (27)$$

Again it is convenient to use (24) to introduce variables β_m which correspond to our antighosts $\beta_z^m(z)$ and which simplify the expression for the measure further. This leads to

$$1 = \int d^{16}\theta d^{16}p d^{16}\lambda d^{16}w d^{10}\xi d^{10}\beta \times \\ \times e^{(i \lambda^\alpha w_\alpha + i p_\alpha \theta^\alpha + i \xi^m \beta_m + \xi^m p \gamma_m w)} (p \beta \beta \beta p) \Omega^{(3)}.$$

where $\beta_{\alpha\beta} = \gamma_{\alpha\beta}^m \beta_m$. As a last step we introduce commuting variables χ_α and κ^α which are the final pairs of (anti)ghosts in our work. The operator $(p \beta \beta \beta p)$ can be written as follows

$$(p \beta \beta \beta p) = \partial_{\chi_\alpha} (p \beta \chi) \gamma_{\alpha\beta}^m \beta_m \partial_{\chi_\alpha} (p \beta \chi) , \quad (28)$$

The integration over χ_α requires the introduction of another delta function $\delta^{16}(\chi)$ in order to make these integrals finite. So we have

$$1 = \int d^{16}\theta d^{16}p d^{16}\lambda d^{16}w d^{10}\xi d^{10}\beta d^{16}\chi d^{16}\kappa \times \\ \times e^{(i\lambda^\alpha w_\alpha + ip_\alpha \theta^\alpha + i\xi^m \beta_m + i\kappa^\alpha \chi_\alpha + \xi^m p \gamma_m w + \beta_m p \gamma^m \chi)} (\kappa^\alpha \gamma_{\alpha\beta}^m \kappa^\beta \beta_m) \Omega^{(3)} .$$

We have thus constructed a formula for the measure

$$\mu(\theta, \lambda; \xi, \beta, \chi, \kappa, w, p) = \\ e^{(i\lambda^\alpha w_\alpha + ip_\alpha \theta^\alpha + i\xi^m \beta_m + i\kappa^\alpha \chi_\alpha + \xi^m p \gamma_m w + \beta_m p \gamma^m \chi)} (\kappa^\alpha \gamma_{\alpha\beta}^m \kappa^\beta \beta_m) , \quad (29)$$

which contains the Fourier transform of all independent pairs $(\lambda, w), \dots (\kappa, \chi)$. The remaining piece $(\kappa^\alpha \gamma_{\alpha\beta}^m \kappa^\beta \beta_m)$ has a very interesting property. It can be identified with the spin 3, ghost number -3 field $\Phi_{zzz}(z) = (\kappa_z^\alpha \gamma_{\alpha\beta}^m \kappa_z^\beta \beta_{zm})$ appearing in the previous section. Notice that we can also make a redefinition of the fields $\lambda^\alpha \rightarrow \lambda^\alpha - i\xi^m (\gamma_m p)^\alpha$ and $\kappa^\alpha \rightarrow \kappa^\alpha - i\beta^m (\gamma_m p)^\alpha$ to recast the measure in a more elegant form

$$\mu(\theta, \lambda; \xi, \beta, \chi, \kappa, w, p) = e^{(i\tilde{\lambda}^\alpha w_\alpha + ip_\alpha \theta^\alpha + i\xi^m \beta_m + i\tilde{\kappa}^\alpha \chi_\alpha)} (\kappa^\alpha \gamma_{\alpha\beta}^m \kappa^\beta \beta_m) . \quad (30)$$

We conclude that we have provided a path integral formula for the tree level measure. We need all ghosts of our formalism to be able to construct the formula (30). We have recovered the operator Φ_{zzz} from a completely different point of view. The exponent in (30) suggests that the ghost current should be changed to $j^{gh} \rightarrow j^{gh} + p_{z\alpha} \theta^\alpha$. Notice that the action $\int d^2z p_{z\alpha} \bar{\partial} \theta^\alpha$ is invariant under the rigid transformations generated by $\oint p_{z\alpha} \theta^\alpha$. The new ghost current j^{gh} has an anomaly equal to $c_j = 6$.

4 Summary and Open Problems

We have constructed a tree level measure for the string field theory formulation of our approach to the superstring. As we shall publish elsewhere, one

can introduce two further BRST charges, such that all terms in (18) except the last one, are BRST exact. In the vertex operator Φ one may then drop all terms except $\lambda^\alpha A_\alpha$. Then we only need retain the following terms

$$A_\alpha \sim a_m(\gamma^m \theta)_\alpha + (\gamma_m \theta)_\alpha (\theta \gamma^m \psi) + F_{mn}(\theta \gamma^{mnt} \theta)(\gamma_t \theta)_\alpha + \mathcal{O}(\theta^5). \quad (31)$$

(These are the only structures which remain in the WZ gauge $\theta^\alpha A_\alpha = 0$.) The action in (10) then yields the complete SYM action in 10 dimensions.

So the following picture emerges. Our completely covariant approach with many more ghosts than only λ^α has linearized the theory to a point where no constraints are any longer present, and no non-covariant canonical approach is needed. Yet, at the end we get rid of all excess ghosts, and recover the formulation with only λ^α . Thus this approach combines the best of the covariant approach with the maximal number of ghosts and the minimal approach with only λ^α . We hope that these ideas will also be effective at the loop level.

There are (of course) many problems left:

- To define physical states we had to restrict vertex operators such that they have “non-negative grading”. The grading is a quantum number one can assign to the ghosts because the underlying algebra is non-semisimple. In [1] we have shown that it is related to homological perturbation theory, but it is desirable to find an extra BRST operator which selects non-negative grading.
- We should calculate the cohomology of the model with J_M^g, J_M^{gh} and J_M^h together with the Koszul quartet. We have found two BRST charges which anticommute, and further conditions involving $B_{zz}(z)$ which remove the dependence on the differences $x^m - x_h^m$, etc, but a complete analysis is in preparation.
- So far we focused only on the left-moving sector *i.e.* the heterotic string, but a complete treatment containing both left-moving and right-moving sectors for all fields, in particular for the WZNW model, is still lacking. Also this we intend to construct.
- The big problem is, of course, the calculation of amplitudes: path integrals, measures and vertex operators. We hope that here our covariant methods will be helpful.

Note added

On the day we submitted this article to the hep-th arXive we heard a seminar by N. Berkovits, who constructed a general expression for the measure in his approach with pure spinor constraints. Although there are differences between his and our approach, for example complex vs. real λ^α ghosts resulting in integrations over 11 instead of 16 λ 's, there is also overlap with the present work.

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